Motif Horizon Models

A ROBUST ASSET ALLOCATION FRAMEWORK FOR INDIVIDUAL INVESTORS AND FINANCIAL ADVISORS
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Introduction

The Horizon Models are the cornerstone of Motif’s asset allocation offering for individual investors and financial advisors. In introducing the Horizon Models, Motif aims to provide a low-cost, diversified asset allocation solution that is built using a rigorous methodology. The models have been implemented as nine Horizon motifs that cover a spectrum of investment horizons and investor risk tolerances. More broadly, the Horizon models provide a robust framework that can be used to develop asset allocation solutions tailored for an investor’s specific financial goals and risk appetite. This white paper describes the underlying philosophy, methodology and implementation of Motif’s Horizon models.

All investors, knowingly or unknowingly, have an asset allocation. While some are more active and allocate capital to a portfolio of funds, others do it while making everyday decisions such as putting money into a savings account, buying a home or investing in the latest IPO. Past research has also shown that over 93% of variation within a portfolio’s returns can be attributed to the allocation to broad asset classes. The selection and weights of broad asset classes can be the primary driver of both returns and risk in an investor’s portfolio. Therefore, the important questions to consider are

i. Is an investor’s allocation to asset classes optimal given their financials goals?
ii. Is the risk profile of the allocation consistent with investor’s appetite for risk?

With the introduction of the Horizon models, Motif hopes to answer the questions posed above.

In developing the Horizon Models, Motif draws upon the Nobel Prize winning research of Harry Markowitz and Modern Portfolio Theory (MPT). While MPT lays the foundation for developing diversified investment portfolios, the methods prescribed under classical MPT have serious limitations. In particular, the use of unreliable forward-looking assumptions for asset class returns, the suboptimal choice of volatility as a measure of risk, and simplistic assumptions around co-movement of asset classes in classical MPT can lead to highly unstable solutions.

Our approach tackles the limitations of classical MPT by employing advances in computational techniques and statistical methods. These methods allow the use of downside risk as a more intuitive measure of portfolio risk, the incorporation of the probability of tail events in the market, and the richer historically observed co-movement of asset classes in the model. Our approach also recognizes the fact that investment horizon and risk tolerance are independent decision parameters. While a longer investment horizon is often equated with higher risk tolerance, Motif deems such an approach sub-optimal. Depending on individual circumstances, two investors with the same time horizon can have very different willingness and ability to handle portfolio downside risk.

We also recognize the fact that firstly the only data that is known with complete certainty is historical data on asset returns. Secondly, a rational investor with no information will choose a portfolio allocation that mirrors the broader market. Therefore, we do not attempt to make any forward-looking predictions, but instead focus on quantifying the information in historical data that can then be used to systematically guide an asset allocation decision based on an investor’s time horizon and risk tolerance. This allows us to develop a rigorous solution to the optimal asset allocation problem while avoiding any subjective predictions or arbitrary model constraints.
A rigorous methodology, to be effective, must be complemented with a low-cost and efficient implementation. The first key aspect of an asset allocation solution is the selection of asset classes. Our choice of asset classes is based on a comprehensive approach i.e., we do not include or omit asset classes based on subjective views. The six asset classes used in the Horizon models - US Equities, International Equities, US Bonds, International Bonds, Real Estate and Commodities – represent the broadest asset classes available to US investors and that have historically provided differentiated and diversified exposure through all phases of an economic cycle.

The second key aspect of an implementation is the choice of investment vehicles that provide efficient exposure to the underlying asset classes. Here, we adhere to the principles of passive indexing and use low-cost exchange-traded funds (ETFs) that seek to track the performance of broad diversified indices that represent each asset class.

As we introduce the Horizon models, we have constructed nine motifs that span from the short-term (1 year) to long-term (15 year) along investment horizon and from conservative through aggressive in terms of risk tolerance. All nine Horizon motifs are implemented using low-cost, passive, best in class ETFs that represent each asset class. The easily customizable motifs allow investors and advisors the flexibility to use the underlying asset allocation model while expressing their own views through stocks and ETFs that they choose.

The final aspect of an implementation has to do with monitoring and rebalancing. The Horizon Motifs are regularly monitored and updated to reflect changes in the market portfolio and observations of asset class returns. Motif employs a well-known ‘time and threshold’ based rebalancing method for this purpose. Motif alerts investors when the weights of the asset classes deviate from pre-defined thresholds and portfolio constructions become sub-optimal. This helps investors keep costs at a minimum as it reduces unwarranted rebalance trades while keeping their portfolio aligned with their goals and preferences.

This rest of this paper lays out the development and implementation of Motif’s Horizon Models in detail. The paper is organized as follows:

- We identify the set of distinct non-overlapping asset classes that best represent the market portfolio
- We next describe improvements on Modern Portfolio Theory (MPT) achieved using advances in computational techniques to develop a risk-reward framework
- We next use the risk-reward framework and apply a rigorous learning process to historical asset returns data to in construct historically optimal portfolios
- We next address the blending of historically optimal portfolios the broad market portfolio to develop Horizon models that cater to a spectrum of investment horizons and risk tolerances
- We next outline the implementation of the Horizon models as motifs through low-cost, best-in-class ETFs, representing each asset class
- Finally, we discuss the monitoring of the Horizon models and their threshold-based rebalancing
Horizon Models: Asset Classes

Selection Criteria

One of the key decisions involved in asset allocation is the selection of asset classes. The selection is closely tied with the investor’s stated goals and risk preferences. For example, an investor looking for income generation is more likely to invest in coupon paying US bonds rather than commodities, which are preferred for long-term inflation protection.

Allocating capital across multiple asset classes can help investors maximize their risk adjusted return. That being said, each additional asset class that is included should add value (e.g. increase return or reduce risk) to the investor’s portfolio. If there is no added value, the investor is merely duplicating exposures already present in the existing portfolio. The asset classes selected should also be investable in an efficient and cost effective way. As a result, the key components of Motif’s asset class selection criteria are:

- Diversification
- Accessibility
- Functional Performance
- Investor preferences

In addition, we consider each asset class’ potential return and sensitivity to various economic scenarios including interest rate movements, economic expansion or contraction, strength of the US dollar and inflation, among others. These factors help to identify key asset classes that can help an investor’s portfolio weather each scenario while seeking to remain on track to realize their goals.

On a broad level, asset classes and their functionality can be bucketed into three main categories:

i. Capital Appreciation
ii. Income Generation
iii. Inflation Protection
Asset Classes

**US Equities**
The United States is the largest and most developed economy in the world with a highly liquid and investable stock market. Our US Equities asset class provides exposure to the corporations that are listed and traded on exchanges in the United States. While the US stock market suffered significantly during the Dot-com bubble of 2000-01 and Financial Crisis of 2008-09, it has recovered well by demonstrating strong growth and economic fundamentals. Our allocation to US Equities seeks to provide investors with a source of long-term capital appreciation as well as inflation protection.

**International Equities**
Countries outside the US account for more than 70% of the global GDP and 65% of the world’s stock market. This, along with the global nature of businesses today, makes a strong case for an allocation to International Equities in an investor’s portfolio. Our International Equities asset class covers developed economies such as Japan, United Kingdom, and Germany etc. as well as emerging market countries like China, India and Brazil. These markets can behave very differently from the US due to their distinct economic characteristics and business cycles. International Equities also provide US investors with the opportunity to participate in certain segments of global markets that are growing faster than those domestically. Overall, International Equities can help investors achieve capital appreciation and inflation protection.

**US Bonds**
The US is the world’s largest bond market with debt issuance from the federal, state and local governments as well as agencies and corporations to fund their operations. Historically, US government bonds have displayed lower volatility compared to equities. In addition, corporate bonds also offer investors the opportunity for higher yields to compensate for their credit risk and liquidity profile. Our allocation to US Bonds provides investors with a diversified, lower risk and steady income stream, compared to assets such as dividend yielding stocks.

**International Bonds**
The International Bond market has grown significantly over the past few decades and stands at almost 1.5 times the US bond market. The International Bonds asset class consists primarily of debt issued by government, non-government organizations and corporations based in developed and emerging countries. International bonds have historically offered higher yields than US bonds due to their credit risk profile. A series of defaults between the 1980s and 2000s serve as justification for these elevated yields. Our allocation to International Bonds seeks to provide investors with income and opportunities for capital appreciation along with diversification benefits at the portfolio level.
**Commodities**
Commodities reflect the prices of natural resources such as energy (oil, natural gas), industrial metals (aluminum, copper), precious metals (gold, silver), agricultural products (soybeans, cotton) and livestock (cattle, hogs). Commodities can serve as a vehicle for inflation protection as raw commodity prices are an input to the Consumer Price Index (CPI). Historically, commodities have had a low correlation to other asset classes that can be used to help obtain diversification across the portfolio. The allocation to commodities can be tax efficient since they do not provide an income stream and can have favorable tax treatment when sold (i.e. taxed as long term capital gains instead of ordinary income).

**Real Estate**
Real estate refers to US real estate investment trusts (REITs) that own, operate and manage residential and commercial properties across the country. REITs are mandated to distribute the majority of their earnings to investors and therefore can have high dividend yields, which can provide increased income. Given that rents and property values move in step with economic growth, REITs can help provide investors with inflation protection and the potential for long-term capital appreciation. Historically, REITs have demonstrated lower correlation to other asset classes and thus may help achieve greater diversification at the portfolio level.

**Notable Exclusion**
As described earlier, Motif’s Horizon Models contain six distinct diversified asset classes that provide investors with exposure to all major investable segments of the global market. One notable exclusion is the Treasury Inflation Protected Securities (TIPS). TIPS were introduced in 1997 by the US Treasury and are a relatively new asset class. Based on recent market data, TIPS account for less than 3% of the US Bond market and an even smaller percentage of the global multi-asset market portfolio.

TIPS by definition, are supposed to protect investors from inflation. However, research has shown that TIPS correlate better with changes in inflationary expectations than the level of inflation itself. While the US Treasury adjusts TIPS’ principal periodically to reflect changes in the Consumer Price Index (CPI), such protection comes at the cost of additional risk. This risk manifests itself in the form of higher volatility that can be attributed to longer duration and increased interest rate sensitivity of newly issued TIPS bonds, most of which have a maturity of eight to nine years.

Another key reason why Motif excludes TIPS from its Horizon Models is their sub-par implementation via ETFs. Investors who hold the underlying TIPS bonds through to maturity are much more likely to be well compensated for inflation risk, compared to those holding an ETF. This is because ETFs constantly roll over their bonds before maturity and thus may not get the optimal benefit of receiving the inflation-adjusted principal back at maturity date. While this practice is prevalent across all bond ETFs, it has a significantly adverse impact on TIPS ETFs, which can make them less effective as an inflation hedge. Instead, investors can be better served and compensated for their risk profile by considering the US Real Estate and Commodities asset classes for inflation protection.
Optimal Asset Allocation: The Classical Approach

Modern Portfolio Theory

Many of the asset allocation decisions such as defining the proportion of each asset in a portfolio, are based on a theory in finance called the Modern Portfolio Theory (MPT). It was developed by Harry Markowitz who later won a Nobel Prize for his work. MPT is a mathematical formulation of the concept of diversification or holding a set of assets that have collectively lower risk than any individual asset. The original formulation of MPT was based on the assumption that expected returns and “risk” characteristics of all the assets are known a priori - the problem then was to determine the weights for each asset class such that the expected returns for the portfolio are maximized while the risk is minimized.

While MPT has been an intuitive and popular method for construction of asset allocation solutions, the MPT problem is somewhat ill posed in its basic form, and raises a lot of questions:

- What exactly is meant by ‘risk’?
- How does one estimate the expected return for an asset?
- Expectation implies a certain probability distribution, and therefore what properties of the probability distributions are needed to make the problem tractable?
- Asset classes such as US Equities and US Bonds have historically shown varying degrees of co-movement in different market conditions – how does one specify the co-movement of asset classes?

Mean Variance Optimization (MVO)

The MPT problem has a well-known solution when the following simplifying assumptions are applied:

- The expected returns for each asset class are known
- The returns for asset classes are normally-distributed (‘bell-shaped’)
- Correlation coefficients among asset-class returns are linear—in other words, the same correlation coefficient applies in both up and down markets.
- Risk is defined as the standard deviation of portfolio returns, and
- The covariance matrix, i.e. the set of correlation coefficients among asset class returns, is known

These assumptions lead to an elegant solution to the MPT problem in the form of a set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. The set of optimal portfolios is called the “efficient frontier” and is usually illustrated as a curve on a graph of expected portfolio returns vs. standard deviation of portfolio returns. This procedure for calculating the efficient frontier is commonly referred to as Mean-Variance Optimization (MVO).
Weaknesses in the Classical Approach

The classical approach to MVO is not without its shortcomings. Over the past decade, practitioners have pointed to several aspects of the MVO framework that can lead to unstable or counter-intuitive asset allocation solutions.

Expected Returns for Asset Classes

A key input to MVO is the expected returns for each asset class. The task of selecting expected asset returns for the model is by no means an easy one. The three commonly used approaches are:

i. **Use historical data on asset returns:**
   Two obvious flaws of complete reliance on historic time-series are scarcity of data (the asset histories are rather short) and the uncertainty about how much asset behavior going forward will resemble the past.

ii. **Use asset returns ‘implied’ by the market**
   The most widely used tool for estimating the market implied set of expected returns is the Black-Litterman model. The Black-Litterman model still requires a non-trivial piece of information—the covariance matrix of asset class returns—which suffers the same deficiency as the estimates of expected returns from historical data, possibly somewhat aggravated by the fact that accurate estimates of the covariance matrix requires even more data than the estimate of returns.

iii. **Use factor models for forward-looking returns**
   The Black-Litterman model, used for calculating the “market implied” expected returns can be combined with the “views” of an investment manager, effectively adjusting return expectations towards various manager biases, be they well-founded or not.

The MVO process is highly sensitive to the expected returns used and even small changes in these estimates result in totally divergent portfolios.

Normal distribution of asset class returns

MVO assumes the normal distribution for asset class returns making the calculation simple and tractable. What is appealing about the normal distribution is that it is intuitive: roughly two thirds of the time, returns are within one standard deviation away from the average return. However, it is unable to account for extreme market events, which occur more often than this model would imply (‘Black Swans’, ‘fat tails’). Figure 1 illustrates that using actual returns for US Equities versus a normal distribution which would imply a less than one-in-million chance of observing the large drawdowns that occurred in October 1987 and September 2008. Many asset classes empirically exhibit return distributions that are skewed to the left of the mean (negative skewness) and that have ‘fatter tails’ (excess kurtosis) than a normal distribution. Accounting for skewness and excess kurtosis has a significant impact on the MVO solution.
Figure 1: Distribution of observed monthly returns for US Equities (1980-2013) and normal distribution fit to the observed data using the sample mean and standard deviation. Events such as October 1987 cannot be explained by the assumption of a normal distribution of returns.

Correlation coefficients among asset class returns

Another limitation of the traditional MVO is that it assumes correlation coefficients among asset-class returns are linear. This contradicts the certain empirically observed asymmetry—asset correlations in bear markets are different from correlations in bull markets. As illustrated in Figure 2, using the historical data on monthly returns for US Equities and US Bonds, the co-movement of asset classes modeled using constant correlation coefficients (top panel) ignores several rich features in observed historical data (bottom panel).
Figure 2: Illustration of the co-movement of asset classes using historical data (1980-2013) on returns for US Bonds and US Equities. The top panel illustrates a model that assumes normally distributed returns with a constant correlation coefficient. The bottom panel shows the co-movement of the same asset classes using a non-parametric density estimate. The irregular shape of the non-parametric density estimate indicates that correlation of US Bonds and US Equities is not constant over time, nor are the returns normally distributed. Note: the axes show log of monthly returns.
Standard Deviation as a Measure of Risk

The MVO framework uses the standard deviation of returns as the measure of risk – the higher the standard deviation, the higher the risk. Standard deviation is a measure of absolute volatility that shows how much a portfolio’s return varies from its average return over time. In a rising market, high volatility can boost the return potential of an investment. Volatility, in other words, does not measure what investors intuitively perceive as risk as illustrated in Figure 3.

![Figure 3: Illustration of limitations of standard deviation / volatility as a proxy for risk using normal distributions of returns. The returns distribution on the left (blue) has a negative mean but lower standard deviation while the one on the right has a positive mean but larger standard deviation. Based on standard deviations, the returns distribution on the right would be considered “riskier”. This implication is not well aligned with an investor’s perception of risk. Varied but consistently positive returns should not considered "riskier" than a set of consistently negative returns.]

Use of Constraints on Asset Class Weights

The MVO process is highly sensitive to the expected return, covariance, and risk estimates used. Moreover, the standard MVO process is quite likely to suggest unintuitive asset allocations that assign non-zero portfolio weights to only two or three asset classes within the complete asset class menu. Many practitioners use ad hoc minimum and maximum portfolio weight constraints to compensate for this and adjust the model-optimized portfolio towards one they perceive as more diversified. In this case MVO produces predefined portfolios and provides little more than a theoretical veneer for an essentially ad hoc process.
Optimal Asset Allocation: The Motif Horizon Approach

The previous section outlined the limitations of traditional mean-variance optimization. The good news is that we can use advances in computational and statistical techniques, learning from financial markets in past decades, and the work of other researchers\(^9\) to develop an objective methodology that builds on the original risk-reward framework of Modern Portfolio Theory and leads to robust asset allocation solutions\(^13,14\).

Asset Returns Distributions

A critical component of optimal portfolio construction is the assumed joint probability distribution of asset returns. Our approach to building an asset allocation model begins by recognizing that the only data not masked by uncertainties are the historical time-series of asset returns. Therefore the assumed distribution of returns must build on what we can learn from history. While historical returns data is not indicative of the future, it is reasonable to assume that certain features of the probability distribution of asset returns that “generated” the historical data persist into the future.

The next challenge lies in actually developing a model for the distribution that “generated” observed asset returns data. What form and shape this distribution has we have no way of knowing with certainty—this is commonly referred to as *distributional uncertainty*. If the general form of the probability distributions was known, or assumed such, the historic data would allow us to estimate various parameters that completely define it. Given the limited amount of data available, the parameters can only be estimated to a certain level of certainty—this is commonly referred to as *parameter uncertainty*. Both the type of distribution and the parameters of a distribution can strongly influence the construction of optimal portfolios. We noted before that both the distributional uncertainty (the simplifying assumption of normal distribution) and parameter uncertainty (return and covariance estimates) in the traditional MVO framework leads to bias and instability in the solution.

We therefore selected a technique that simultaneously deals with both distributional and parameter uncertainties. The *Kernel Density Estimator*\(^15\) is a nonparametric statistical technique that involves approximating a hypothesized probability density function from observed data. We apply the Kernel Density Estimator to more than 30 years of asset class returns data ranging back to 1980. The kernel density thus built on historical data is effectively the posterior probability distribution of asset class returns, conditional on the real data on asset class returns. As detailed in the following sections, the kernel density can be used to estimate portfolio metrics such as return and downside risk, which then neatly fit into a risk-reward framework for computing “optimal” portfolios.
Portfolio Return & Risk

Figure 4: An illustration of the estimating the probability density function for asset class returns using the Kernel Density Estimator (in blue) and using the assumptions of normal density (in orange). This graph uses the monthly return for US Equities for the period 1980-2013 and a log-scale is used the vertical axis to highlight the differences between the two estimates. The kernel density estimator has the ability to model the extreme market events as seen in the left fat tail of the distribution.

The two key metrics necessary for an informed portfolio allocation decision are:

**Estimating expected portfolio returns**
There has been an ongoing debate in financial circles for decades on whether the arithmetic mean or the geometric mean of returns is more appropriate for assessing long-term expected returns. There is a case to be made in favor of geometric mean of monthly returns because arithmetic mean tends to overstate the practically achievable cumulative portfolio return. Practitioners also favor the geometric mean because it is correct for measuring realized compound return\(^{16}\). Therefore, we use the geometric mean computed from the joint distribution of asset returns as the measure of long-term expected returns.

**Measuring portfolio risk**
We have discussed the deficiencies of standard deviation of returns as a measure of investor risk. Various alternatives have been proposed\(^ {13, 14} \) that, unlike standard deviation, do not penalize large positive returns. These metrics attempt to convey the idea that the magnitude and probability of a loss is what constitutes investment risk. We have adopted one of these measures called Conditional Value-at-Risk
(CVaR) as the measure of portfolio risk. CVaR equals the average loss over all possible scenarios that see portfolio returns below a certain pre-set level (e.g. average loss on the bottom 5% of possible returns).

**Figure 4:** Illustration of Conditional Value-At-Risk (or CVaR) as a measure of risk. The top panel shows an asset returns distribution that is approximately normal but has a large negative fat tail that corresponds to negative surprises. The bottom panel shows a similar asset returns distribution but with a positive fat tail that corresponds to large positive surprises. The standard deviation for both distributions is identical but the CVaR (average return for the 5% worst cases) is significantly more negative for the distribution in the top panel. Standard Deviation is unable to differentiate between portfolios that have large upside surprises and large downside surprises.

**The Efficient Frontier**

We combine the asset returns distribution and measures of risk and return as described above to develop a set of optimal portfolios each of which has the highest geometric returns for a given level of downside risk (CVaR). The calculations of geometric mean and CVaR for a portfolio is performed using extensive re-sampling from the kernel density – a computationally intensive technique but one that can now be handled using the capabilities of a modern desktop processor. A genetic algorithm seeking to maximize geometric average of returns, while simultaneously minimizing the risk (in our case the CVaR measure) is then used to develop the set of optimal portfolios. This set of optimal portfolios is akin to the efficient frontier used in the MVO framework – and represent the best risk-reward tradeoff for investors seeking to invest purely based on learning from historical data.
Figure 5: The efficient frontier (in blue) of optimal portfolios, i.e., portfolios with the highest geometric mean return for a given level of downside risk, developed using the rigorous learning process applied to historical returns data. The graph also shows the individual asset classes. Unsurprisingly, single asset class portfolios do not lie on the efficient frontier and can have both their risk and return improved through diversification.
Incorporating risk tolerance and horizon

While the efficient frontier, as seen in Figure 5, helps identify the optimal portfolios within a risk-reward framework, investors have varying needs and select the optimal portfolio based on their investment horizon, and risk tolerance. While longer investment horizons at times are equated with higher risk tolerance, our approach treats these as independent decision parameters that cannot be substituted for each other.

**Investment Horizon**

Investment horizon or the time until a cash flow need shape investor attitudes towards short-term risk. While an investor with an impending cash flow need within the next year will be more sensitive to downward moves in their portfolio each month, an investor with a 15-year horizon will have a higher ability to withstand monthly downside risk and will be concerned about portfolio performance over a longer window e.g. 1 year. It is important to make the distinction between horizon and risk tolerance here since each investor with a 15-year horizon still has a unique tolerance for risk over that 1-year monitoring period.

We incorporate investment horizon by developing a distinct efficient frontier for short, medium, and long-term investors. Each frontier is developed using the same methodology described in the previous section, with the key difference being the aggregation window for historical data used in each case. We use 1, 6, and 18-month aggregation of historical data for short-, medium-, and long-term horizons – these correspond to reasonable risk monitoring windows for investment horizons of 1, 5, and 15 years respectively.

**Risk tolerance**

Risk tolerance is typically defined as an investor’s willingness to trade a relative decrease in expected returns for a relative decrease in the acceptable level of risk. Following commonly accepted practice, we consider the range of downside risk (CVaR) spanned by the optimal portfolios on the efficient frontier. For each of the three distinct efficient frontiers (where each efficient frontier is characterized by a unique horizon), we assemble the optimal portfolios on the frontier into three groups - “conservative”, “moderate” and “aggressive” respectively representing low, moderate, and high downside risk.

The three risk tolerance levels for each of three investment horizons thus result in nine portfolios. Collectively, these nine broad asset class portfolios span a broad range of investor preferences for risk tolerance and investment horizon. Within a Bayesian framework, each of these portfolios represents the optimal risk-return trade-off conditional on the historical data on asset class returns.
The Investable Market Portfolio

The methodology, described thus far, helps develop optimal portfolios based on systematic learning from historical data. We have thus far not included any subjective forward-looking predictions in our methodology. However, there remains an objective way of incorporating the market’s view on asset class returns – which is by studying the broad investable market portfolio itself.

When an investor has no information on the probability distributions of relative returns of various asset classes, the most rational capital allocation for the investor would be an equal amount for every dollar already invested by the broad market (i.e. a capital-weighted allocation). While many practitioners that employ the Black-Litterman model also use the market portfolio to obtain market-implied returns, our methodology unlike the Black-Litterman model does not need estimates of a covariance matrix nor the computations to determine actual values for market implied returns. We instead use the capitalization-weighted market portfolio as the least information or maximum-ignorance portfolio for rational investors. Rational investors only alter their asset allocation from the market portfolio based on quantifiable information.

![Figure 6: The capitalization-weighted investable market portfolio for US investors](image)

Learning from Historical Data: The Information Theory Approach

What does a rational investor do when they come into information and how do they incorporate it into their asset allocation decision? Information increases the rational investor’s state of knowledge about the probability distributions of asset returns. This creates an optimization problem where the information may force the new portfolio to deviate from the market portfolio. The exact ‘direction’ of such a deviation will depend on the both the information and on the investor’s preferences (investment horizon and risk tolerance). When the information consists of learning from historical asset returns data, a framework is needed to quantify how much new information is contained therein and, consequently, by
how much the portfolio that takes it into account the information should be allowed to deviate from the market portfolio.

Such a framework to quantify the information can be developed using information theoretic methods\textsuperscript{17, 23}. Within the Information Theory (IT) framework, “information” reflects the decrease in ambiguity regarding a process i.e., when we are ignorant about a process, whatever reduces the bounds of possibility, or concentrates the probability of possible outcomes, informs us. The IT framework is well-suited for our optimization problem since “information” gleaned from historical data is what induces rational investors to change their beliefs in a way that is prompted, but constrained, by the new information.

Since information is defined as the reduction in uncertainty, IT’s formalism for quantification of information is based on the measure of uncertainty called entropy. Entropy in IT is a measure of uncertainty of a random variable and is akin to the notion of entropy as a measure of disorder in thermodynamics. Formally, for a probability mass function, $p$, the information content of an outcome, $i$, is given by

Using Equation (1), the definition of entropy in IT as introduced by Shannon\textsuperscript{17} reflects the expected informational content of an outcome and is defined as

$$H(p) \equiv \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i} = - \sum_{i=1}^{M} p_i \log_2 p_i = E \left[ \log_2 \left( \frac{1}{p(X)} \right) \right]$$

for the random variable $X$ that has $M$ finite states.

Similarly, relative entropy is a measure of “deviation” of uncertainty between two probability distributions. The relative entropy or cross entropy between two probability mass functions $p$ and $q$ is given by

The relative entropy in Equation (2) provides a measure for the gain in information resulting from the additional knowledge in $p$ relative to $q$. Relative entropy in IT is often used as the tool for updating prior probabilities (beliefs) to posterior (post-data) probabilities when new information becomes available.

The formalism of relative entropy may be employed to quantify (a) the information content in historical asset returns data and (b) the information used by an investors when their portfolio deviates from the market (or maximal ignorance) portfolio. We compute the information content of historical data using Equation (3), the probability density function or $p$ used here is the one developed using the kernel density estimate approach described earlier. The “information used” when deviating from the market portfolio are also calculated using Equation (3) since the market portfolio weights represent the prior and the final portfolio weights represent the posterior probabilities respectively. The two measures – historical information content and information used – can then be combined to develop the optimal asset allocation for an investor’s specific risk tolerance and horizon.
The Optimal Asset Allocation

This finally leads us to the optimal asset allocations that blend the market portfolio and the optimal portfolio based on historical data. As described above, the equilibrium market portfolio represents the best possible allocation with no additional information available to the investor. On the other hand, the historically optimal portfolios systematically account for what an investor may learn from observed asset returns data.

Neither of the views, the market portfolio or the historically optimal portfolio, can be adopted as optimal with certainty. In order to quantify the degree of belief that one of them is more likely to be a “true” optimal portfolio, we develop a blend of the two portfolios. We select the blending coefficient in a manner such that the information content (i.e., reduction in entropy within the Information Theory framework) of the blended portfolio matches the information content of the historical asset returns data that was used to calculate the historically optimal portfolio. For instance, a reduction in entropy of the blended portfolio that exceeds the information content of available historical data, would imply that we put more faith in historical data than it warrants.

We use this systematic blending of historically optimal portfolios and the market portfolio to arrive at the final optimal weights for asset classes. Each historically optimal portfolio has a distinct blending coefficient. For example, the blending coefficient for the Conservative Long-term optimal portfolio differs from that for the Conservative Short-term optimal portfolio. The blended portfolio is allowed to deviate from market Portfolio in the direction of the historically optimal portfolio, but by no more than the information contained in historical asset returns data. These blended portfolios represent the Horizon model’s asset allocation solution. No further post-processing, arbitrary constraints or artificial diversification are required.
Putting it all together: Horizon Motifs

As detailed in the sections above, the methodology used to construct the Horizon Models can be used to tailor an asset allocation for an investor’s investment horizon and risk tolerance. The Horizon Motifs, that are built and rebalanced using the Horizon asset allocation model, span a wide range of investor preferences (i.e., investment horizon and risk tolerance). Collectively, these nine motifs range from short-term (1 year) to long-term (15 year) along the investment horizon dimension and from conservative through aggressive on the risk tolerance dimension provide good starting points for an investor seeking an asset allocation solution.

The following figures illustrate the allocation to asset classes in the nine Horizon motifs. For a given investment horizon, as one moves up the risk tolerance scale, the allocation progressively moves from US Bonds and International Bonds to historically riskier asset classes (i.e. US Equities, International Equities, Commodities and US Real Estate). The changes in the asset class weights with the investment horizon for a given level of risk tolerance are not as dramatic, but there is a discernible shift towards International Bonds and International Equities with an increase in investment horizon.

![1 Year Horizon Motifs](image)

*Figure 7: Allocation to asset classes in the short-term (1 Year) Horizon motifs*
Figure 8: Allocation to asset classes in the medium-term (5 Year) Horizon motifs

Figure 9: Allocation to asset classes in the long-term (15 Year) Horizon motifs
Implementation

While developing a distinct asset allocation model to suit investor needs is key, implementing it in a cost efficient way is equally important. Motif utilizes passive, low-cost index based ETFs to represent each asset class in its Horizon Motifs.

Historically, mutual funds have been the preferred choice of investment vehicle for individuals as well as financial advisors. This was primarily driven by the wide variety of offerings, performance track-record and lack of reasonable alternatives. This coupled with investor belief in active management helped the mutual fund industry gather over $13 trillion in assets\(^1\), while charging management fees, sales load, redemption fees, in order to manage the funds according to the parameters defined in its prospectus. However, with the average fees being in excess of 1% annually, mutual funds can be cost prohibitive for the long term, as they eat into investor returns\(^1\). Additionally, research has shown the most actively managed funds under-perform their benchmark and those that outperform in one year, often underperform in the next.

The evolution of the financial services industry over the past decade and introduction of ETFs has dramatically altered the investment landscape. Unlike mutual funds that are priced by their net asset value at the end of the trading day, ETFs are priced in real-time and traded throughout the day. While ETFs also charge a management fee and are subject to commissions when purchased by a broker, their average fees of around 0.60% are much lower than mutual funds\(^1\). Coupled with their low fee structure, ETFs can be a more transparent and more liquid investment vehicle for investors. This has helped ETFs gather over $1.3 trillion in assets under management at a rapid pace\(^1\). In fact, there are over 1400 ETFs in the US that cover all major asset classes, sectors and industries, making it easier for investors to implement investment solutions with greater control and flexibility.

In order to implement our asset allocation solution in a low-cost way while providing investors with the truest exposure to the underlying asset class, Motif scans the universe of US exchange listed ETFs and selects the ‘best in class’ within each asset category. We do this by utilizing a variety of parameters in our selection criteria, as defined below:

**Costs**

Expense ratios represent the ETF’s cost to run the fund and include both operational and trading expenses. Lower expense ratios can be favorable for investors as they detract less from aggregate returns achieved.

**Liquidity Profile**

The liquidity profile of an ETF showcases its ability to absorb an investor’s order with minimal market impact. We use asset under management (AUM), shares traded and dollar volume to assess ETFs. A large AUM base demonstrates the ETFs ability to attract investor interest and can be indicative of track record and ability to provide the required exposure with low tracking error. Trading volumes are often inversely correlated with bid-ask spreads and low trading volumes can result in a large spreads, leading to higher costs. Therefore ETFs with high AUM and volumes are preferable.
Quality
There are often multiple ETFs providing the same style of exposure. This makes it necessary to invest in the one with the highest quality. Motif believes this is best demonstrated by analyzing the number of holdings, concentration and tracking error with regards to the fund’s benchmark. Motif favors ETFs that hold similar number of securities as the underlying benchmark and have lower concentration compared to its peers. Additionally, ETFs that have demonstrated an ability to track their benchmark as closely as possible are considered the best in class.

Credit Ratings
These refer to the credit worthiness of the ETF’s holdings as rated by independent rating agencies. Credit ratings are a reflection of the holdings and their default risk. This generally applies to fixed income ETFs utilized for the US and International bond asset class. Motif gives preference to ETFs that have the highest credit rating within in peers, while providing the same underlying exposure.

Time & Threshold Based Rebalancing
Once an investor selects an appropriate Horizon motif for their investment portfolio, it is important for investors to monitor and review their portfolio to ensure that it reflects changes in their goals, cash flow needs and risk tolerance. The Motif platform allows investors to easily implement these changes by customizing their existing Horizon Motif or by selecting a different Horizon motif that better suits the investor’s new goals and needs.

A second equally important reason to update the motif is market movements. Market movements can lead asset class weights to deviate from their initial allocation. While daily changes may be small, they can add up over time. A large deviation can lead to a sub-optimal portfolio whose risk-return characteristics are significantly different from those of the individual investor.

The primary goal of a rebalancing strategy is not to maximize returns, but rather minimize risk relative to the target asset allocation. Choosing the right rebalancing strategy is almost as important as selecting the appropriate asset allocation model as it impacts the risk-reward profile of the portfolio. An investor with a higher risk appetite may not be as sensitive to risk relative to the target allocation and rebalance once every few years. As a result, the portfolio may become concentrated in assets that have gone up in value and not provide the intended diversification. Alternately, an investor who wishes to closely align with the target allocation may rebalance the portfolio frequently, leading to high commissions and trading costs, which in turn detract from returns and have tax consequences as well.

While industry practitioners advocate rebalances on a fixed monthly, quarterly or annual schedule, this approach may not be the most efficient one. Research has shown that the continuous rebalance of a 60% equity / 40% bond portfolio on a monthly schedule can detract as much 2.7% in annual returns due to portfolio turnover, without significant risk-return benefits compared to other frequencies.
In order to address these concerns, Motif’s Horizon Models employ a time and threshold based rebalance strategy. The motifs are monitored for deviation from target weights (i.e. portfolio turnover required to maintain target allocation on a monthly basis). If the deviation hits a pre-defined threshold at the scheduled month-end date, the portfolio will be automatically rebalanced back to the target allocation. Movements within the month are not considered for threshold breaches, since daily monitoring and rebalancing can add to the maintenance cost of the portfolio. Taking the same 60% equity / 40% bond portfolio and applying a 5% threshold on a quarterly level can reduce the cost to just 1.7% of total annual returns and the number of rebalances by a factor of 20, while coming ahead on aggregate risk-return metrics.

Motif’s time and threshold based rebalance approach benefits investors through:

- Objective, rules-based rebalancing allows for investors to maintain discipline and reduce unintended reactions based on emotions
- Reducing unnecessary rebalances when the portfolio allocation is consistent with investor goals
- Allowing investors to buy securities that have depreciated in value and causing a deviation from the target weight while selling those that have gone up (i.e. systematic buy low and sell high).
- The ability to tailor the rebalance frequency to an individual investor’s needs i.e. a risk-averse investor may lean towards setting a low threshold while one with a higher appetite may be okay with larger deviations from the target allocation.

**Conclusion**

Motif’s Horizon Models serve as the foundation for a multitude of tailored wealth management strategies. These models provide a robust framework for building tailored asset allocation models for an investor’s core portfolio needs. The methodology used here draws from the principles of Modern Portfolio Theory but also systematically addresses the weaknesses in the traditional approach and avoids any subjective assumptions. This rigorous methodology coupled with the choice of broad non-overlapping asset classes and low-cost ETFs, delivers an institutional-grade asset allocation solution for use by investors and financial advisors.
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About Motif Investing

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